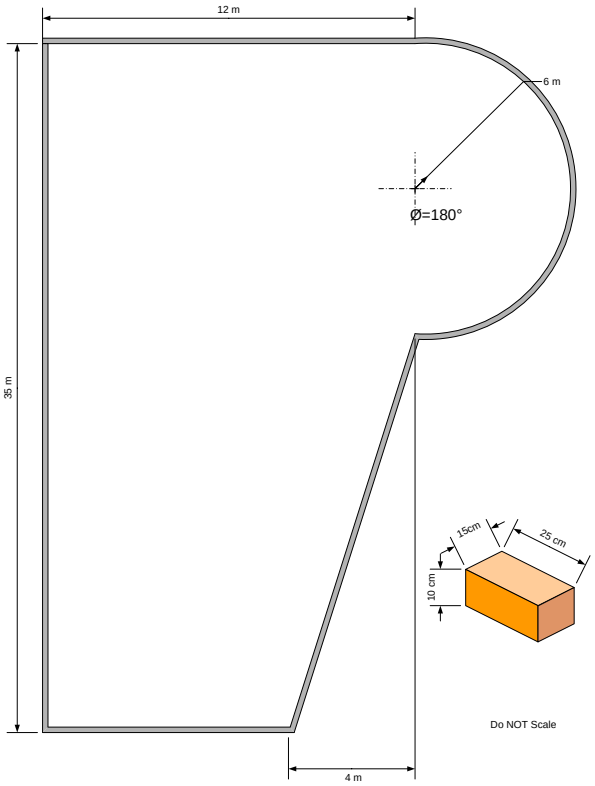
A Model Answer to the Cellar Problem

This work has been set to see if you can:

- break down a complex problem into its constituent parts;
- solve each constituent part;
- use the answers to each part to solve the whole;
- set your work out in a clear and concise manner;
- apply the knowledge that you already have.

With these objectives in mind, below is a model answer for the cellar problem so that you can compare your work to find the answers. For clarity, calculations for the most part have been done away from the answer, but you would need to include them.

What do we know?



The cost each brick is £0.17. A pallet contains 1000 bricks. You can purchase a pallet of bricks at a 7.5% discount to purchasing the individual bricks.

The walls must be 8 feet high.

The ratio of sand to cement in the concrete that you use to glue the bricks together is 7:2. Altogether, you need 370kg of concrete. Cement comes in 20kg bags, each costing £8.75. Sand comes in 10kg bags, each costing £3.10.

The cost of the carpet is £27.95 per square metre. The cost of fitting the carpet is £160. There are reductions available depending on the cost of the carpet.

A brief plan

From the plan of the cellar, we have some of the dimensions but we will need to calculate extra dimensions in order to be able to solve the problem.

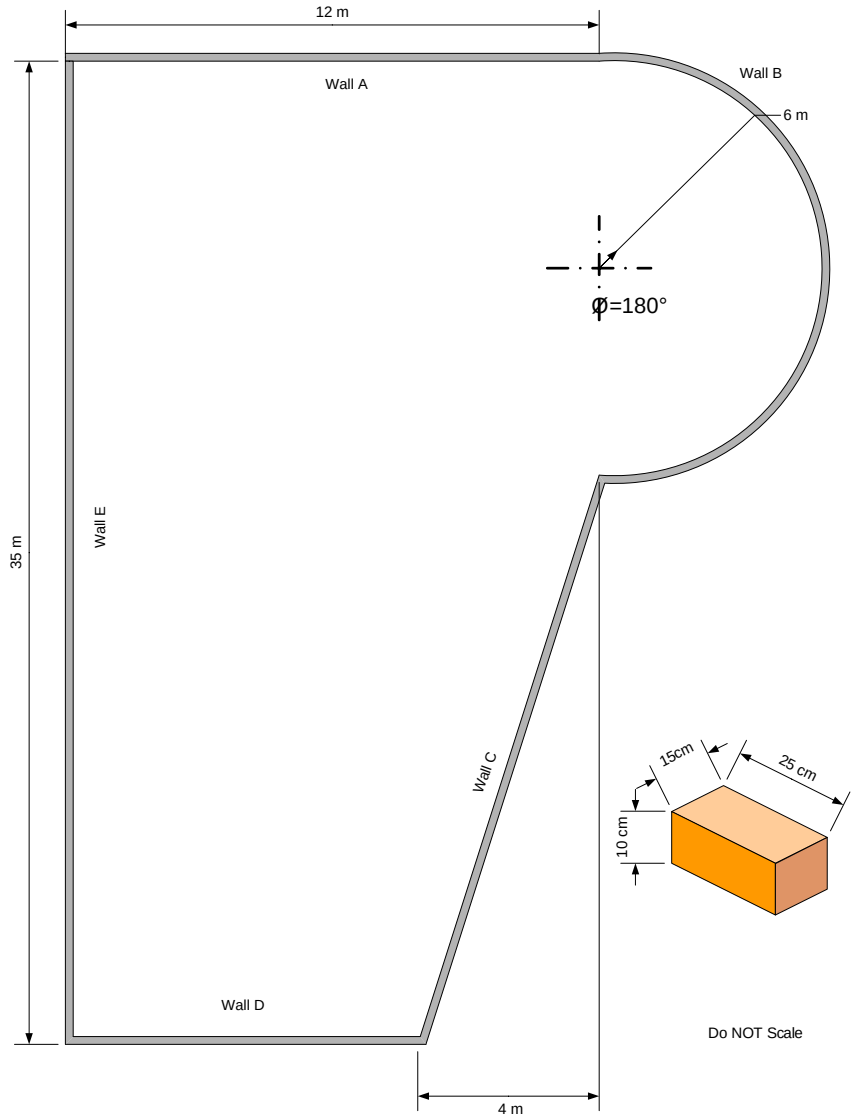
We will need to calculate the length of the wall on the curved section, the adjoining diagonal section and the base wall on the plan.

We will need to split the plan into sections to calculate the area of carpet required. We will need to calculate the cost of the carpet and then the total cost including any discounts and the fitting of the carpet.

We will need to convert the height of each wall into metric units.

We will need to calculate the number of bricks needed for each wall and the cost of the bricks. We will need to calculate the cost of building the wall.

To calculate the extra dimensions of the cellar walls.



Wall A is 12m long.

Wall B needs calculating.

Wall C needs calculating.

Wall D needs calculating.

Wall E is 35m long.

To calculate the length of Wall B:

The circumference of a circle is calculated by circumference, $C = 2\pi r$ where r is the radius.

$$C = 2\pi r$$

$$= 2\,(3.142) \times 6$$

$$= 12(3.142)$$

$$= 37.704\ m$$

This would be the length of the circumference round a whole circle. However, we have a semi-circle and so we will need to halve this value.

$$Length\ of\ wall\ B = \frac{1}{2} \times 37.704$$

$$= 18.852m$$

To calculate the length of Wall C:

The only length we have relating to wall C is the base. This is 4m and will prove useful if we can find the height.

To the calculate the perpendicular height of Wall C:

Wall C_{Height} = Wall E - (2×radius of Wall B)

= 35 - (2 × 6)

= 35 - 12

= 23m

To calculate the length of Wall C:

We need to use Pythagorus Theorum which states that in a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides. So in this case, we have:

$$C_{Height}{}^2 + C_{Base}{}^2 = C^2$$

$$23^2 + 4^2 = C^2$$

$$529 + 16 = C^2$$

$$C = \sqrt{545}$$

= 23.345m (to 3 decimal places)

To calculate the length of Wall D:

Wall D = Wall A - Wall C_{base}

= 12 - 4

= 8m

Summary of Information Calculated:

Wall A is 12m long.

Wall B is 18.852m long.

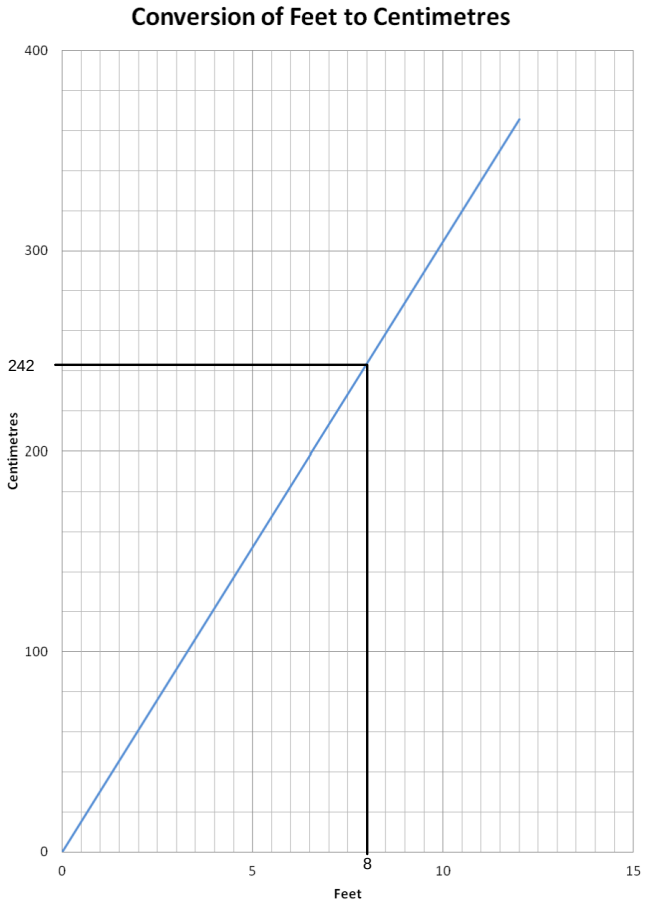
Wall C is 23.345m long.

Wall D is 8m long.

Wall E is 35m long.

To calculate the cost of the bricks:

To convert height of wall (8 feet) to metric measures.



From the conversion graph, 8 feet equates to about 2.42m.

To calculate the number of bricks high in each wall:

$$Number\ of\ bricks\ high\ in\ wall = \frac{height\ of\ wall}{height\ of\ brick}$$

$$Number = \frac{2.42}{0.1}$$

Number of bricks high = 24.2 bricks

However, as we don't want to be cutting every brick along their length, round up to 25 bricks high. (The cement may account for the fifth of a brick, but better being safe than sorry.)

To calculate the number of bricks for wall A:

$$Number\ of\ bricks\ along\ length = \frac{Length\ of\ wall}{Length\ of\ brick}$$

$$Number = \frac{12}{0.25}$$

= 48 bricks along.

Total number of bricks = Number of bricks along length × Number of bricks high

= 48 × 25

= 1200 bricks.

To calculate the number of bricks for wall B:

$$Number\ of\ bricks\ along\ length = \frac{Length\ of\ wall}{Length\ of\ brick}$$

$$Number = \frac{18.852}{0.25}$$

= 75.408 bricks along.

Total number of bricks = Number of bricks along length × Number of bricks high

= 75.408 × 25

= 1,885.2 bricks.

This rounds up to 1886 bricks.

To calculate the number of bricks for wall C:

$$Number\ of\ bricks\ along\ length = \frac{Length\ of\ wall}{Length\ of\ brick}$$

$$Number = \frac{23.345}{0.25}$$

= 93.38 bricks along.

Total number of bricks = Number of bricks along length × Number of bricks high

= 93.38 × 25

= 2,334.52bricks.

This rounds up to 2,335 bricks.

To calculate the number of bricks for wall D:

$$Number\ of\ bricks\ along\ length = \frac{Length\ of\ wall}{Length\ of\ brick}$$

$$Number = \frac{8}{0.25}$$

= 32 bricks along.

Total number of bricks = Number of bricks along length × Number of bricks high

= 32 × 25

= 800 bricks.

To calculate the number of bricks for wall E:

$$Number\ of\ bricks\ along\ length = \frac{Length\ of\ wall}{Length\ of\ brick}$$

$$Number = \frac{35}{0.25}$$

= 140 bricks along.

Total number of bricks = Number of bricks along length × Number of bricks high

= 140 × 25

= 3500 bricks.

To calculate the total number of bricks:

Total Number of Bricks = A + B + C + D + E

= 1,200 + 1,886 + 2,335 + 800 + 3,500

= 9,721 bricks

To calculate the cost of the bricks:

Cost of bricks = Number of bricks × Cost of brick

= 9721 × 0.17

= £ 1,652.57

However, there is a 7.5% reduction in the cost of bricks if I purchase them by the pallet (1000 bricks).

Cost of 9000 bricks = Number of bricks × Cost of brick × 92.5%

= 9000 × 0.17 × 92.5%

= 1530 × 92.5%

$= 1530 \times \frac{925}{1000}$

= £1,415.25

The question is whether it is cheaper to purchase a pallet of 1000 bricks at 7.5% discount or purchase 721 individual bricks with no discount.

Cost of 721 bricks = 721 × 0.17

= £122.57

Cost of 1000 bricks = 1000 × 0.17 × 92.5%

= 170 × 92.5%

= £157.25

So the cheaper option is to purchase the individual bricks.

The total cost of the bricks = 1415.25 + 122.57

= £ 1,537.82

To calculate the cost of concrete:

We need 370kg of concrete. The ratio of sand to cement is 7:2.

To calculate the denominator:

Denominator = sum of parts

= 7 + 2

= 9

To calculate the amount of sand required:

$$Sand = \frac{7}{9} \times 370$$

$$= 287.7778\ kg$$

As sand is sold in 10kg bags for £3.10, we will need 29 bags of sand at £3.10 each.

Cost of sand = 29 × 3.10

= £89.90

To calculate the amount of cement required:

$$Cement = \frac{2}{9} \times 370$$

$$= 82.2222\ kg$$

As cement is sold in 20kg bags for £8.75, we will need 5 bags of cement at £8.75 each.

Cost of cement = 5 × 8.70

= £43.50

To calculate the total cost of materials to build the cellar:

Cost = bricks + cement + sand

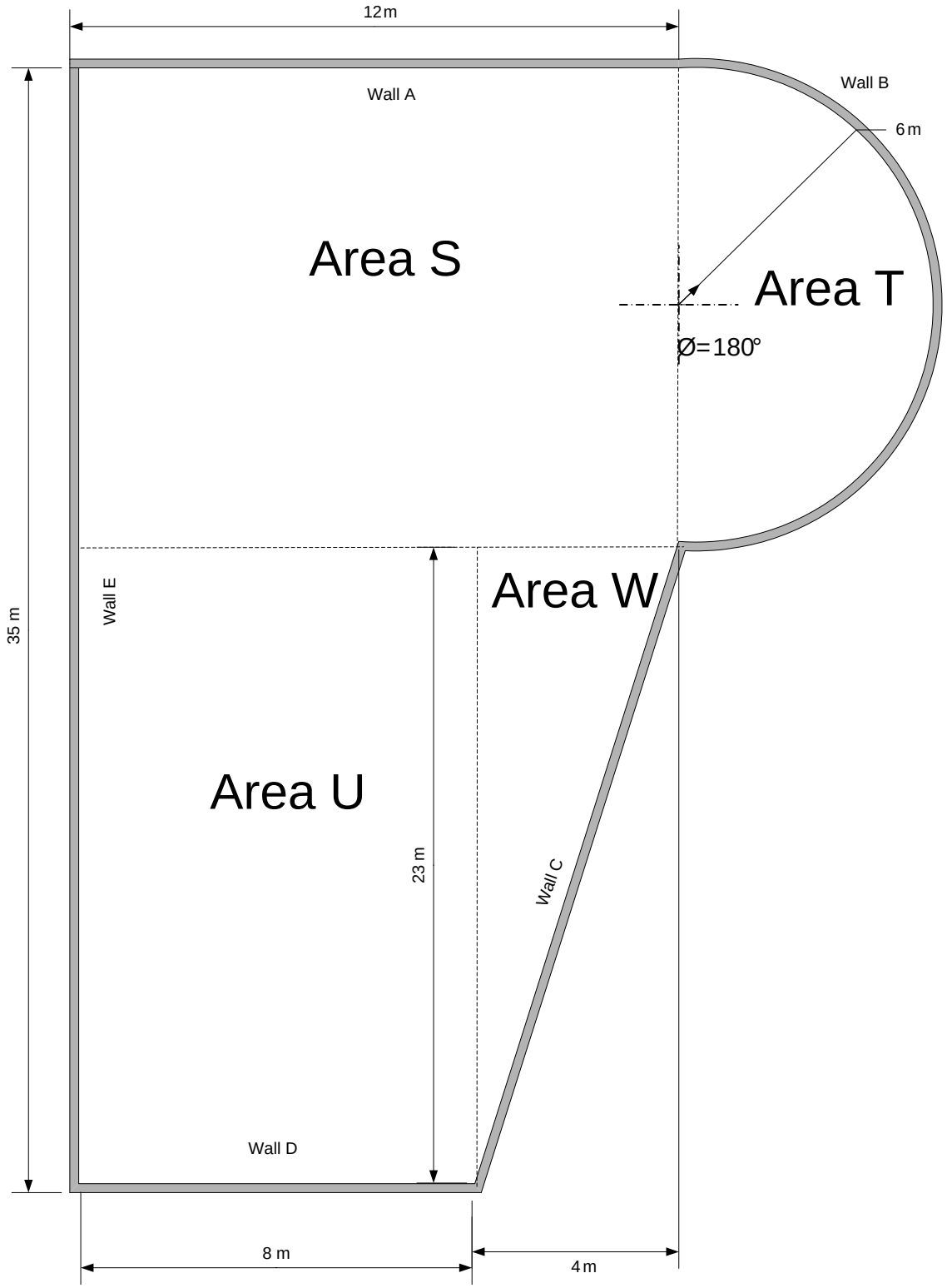
= 1,537.82 + 43.50 + 89.90

= £1,671.22

To calculate the cost of the carpet in the cellar:

First, we need to split the floor area into different sections so that we can calculate the cost of each section.

This is best done on the diagram.



To calculate the area of Area S:

$Area_s$ = $length_s \times breadth_s$

= 12 × (2×6)

= 12 × 12

= 144 m^2

To calculate the area of Area T:

$Area_T$ = $\pi \times radius^2$

= 3.142 × 6 × 6

= 3.142 × 36

= 113.112 m^2

To calculate the area of Area U:

$Area_U$ = $length_U \times breadth_U$

= 23 × 8

= 184 m^2

To calculate the area of Area W:

$Area_W$ = ½ × $base_W \times height_W$

= ½ × 4 × 23

= 46 m^2

To calculate the area that needs carpeting:

Area = $Area_S + Area_T + Area_U + Area_W$

= 144 + 113.112 + 184 + 46

= 487.112 m^2

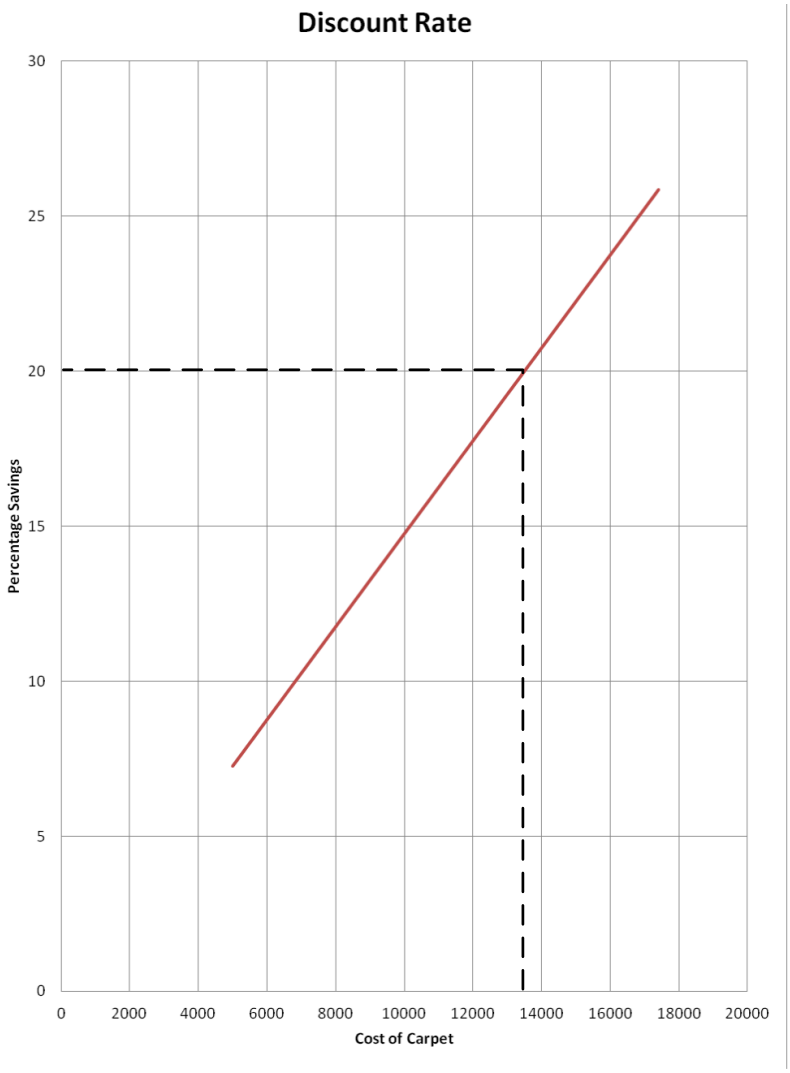
To calculate the cost of the carpet:

Cost = Area × Cost per m^2

= 487.112 × 27.95

= £13,614.78

To calculate the discount on the carpet:



The discount on the carpet is approximately 20%.

To calculate the cost of the carpet:

Actual Cost = Prediscounted cost × 80%

= 13,614.78 × 80%

= £10,891.82

The total cost of the cellar	= Cost of building materials + Cost of Carpet + Fitting of Carpet
	= 1,671.22 + 10,891.82 + 160
	= £12,723.04